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LINCOLN LABORATORY

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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CONTROL SYSTEMS

ABSTRACTS OF RESEARCH STUDIES

MARCH 1963

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ABSTRACTS OF CONTROL SYSTEMS RESEARCH STUDIES

This report is a compilation of abstracts of papers recently produced in the Control Systems research studies at Lincoln Laboratory.

The Control Systems studies form one part of Lincoln Laboratory's General Research Program. The other major parts of that program conduct research in solid state physics, computers, communications, radar, radio physics, and space surveillance.

The Control Systems studies consist of research on critical design problems occurring in electronic systems, particularly in their information gathering, transmission, and processing functions. The many activities of Lincoln Laboratory, in advanced radar, communication, and information processing systems, provide an exacting and stimulating environment for these research studies.

Estimation and Control Theory

Most of the systems with which the Laboratory is concerned, such as in satellite communications, ground control for Project Apollo, or missile defense, have a strong space orientation. Here the critical system functions are tracking and/or dynamic control of space vehicles. These tasks demand advances in the theory and practical application of statistical estimation and extremal control techniques to real-time system operations. One part of the Control Systems research is devoted to developing basic theory and advanced computational techniques for such applications.

The estimation and control studies are producing important theoretical results in optimal trajectory estimation; minimum-time and minimum-energy control procedures; and adaptive techniques for prediction of random time series or for signal detection. The continuing program will emphasize optimum control with noise-corrupted data and efficient computational techniques for real-time trajectory estimation.

Digital System Design

Major elements of electronic systems, such as radars, communication links, and general-purpose digital computers, are continually being brought into closer interaction in advanced, real-time applications. The Laboratory's Millstone space tracking radar, with its on-line computation by the CG24 digital computer, is an early example of such close coupling. Again, in communication systems, the development of automatic digital coding/decoding methods, the use of active and passive satellite relays, and the need for computer control of heavy communication

traffic, are removing the usual distinctions between radar, computer, and communication elements. At the same time, developments in information theory and related doctrines have emphasized this unifying trend in systems design, analysis, and evaluation.

One result of the trend is the growing importance of special-purpose digital machines to meet the increasing data rate requirements at the system interfaces. Fortunately, these important digital design problems are amenable to application of the new techniques for optimum (and computer-aided) design. A second part of the Control Systems research is directed toward advancing the methods for efficient design of special-purpose machines to meet critical real-time information processing needs.

Automatic procedures for design of minimal logical networks have been developed and applied to critical systems problems by design of special-purpose digital computers. The computer-programmed design facilities include optimum minimization routines for single and multi-output combinational networks and efficient algorithms for minimizing sequential machines. These tools have been applied successfully in such areas as design of an advanced digital communications decoder, a digital processor of radar monopulse signals, and a computer for real-time radar ranging of space vehicles using a chain-code, binary pulse waveform. The continuing program will stress the extension of the automatic design procedures to more complex data handling problems and the use of finite field arithmetic and other novel design concepts to reduce machine complexity.

Information Systems

Perhaps the most significant characteristic of highly integrated, electronic surveillance and control systems is their over-all functional complexity. The specification of system requirements, the orderly design and evolution of the system, and effective evaluation of the over-all system performance, all require new understanding of the principles of system organization and methods for effective system control. In computer-coordinated systems most of these problems are reflected in the design of complex real-time computer programs. A third part of the Control Systems research is concerned with modeling and analysis of such complex system operations and development of system control procedures.

Techniques have been developed for effective organization and specification of large computer programs. For store-and-forward communication networks, the effects upon message delays of routing procedures, priorities, and network structure have been investigated by analysis and digital simulation. Future work will emphasize the modeling and analysis of systems combining data transmission with automatic data processing.

The following staff contributed to the research program:

Dr. A. W. Armenti
Dr. M. Athans
T. C. Bartee
Dr. P. L. Falb
L. A. Gardner, Jr.
Dr. L. Kleinrock
Dr. H. J. Kushner
R. T. Lacoss, University of California (Consultant)
M. M. Marill
Dr. R. C. Norris
Prof. W. W. Peterson, University of Florida (Consultant)
Dr. R. T. Prosser
Prof. D. J. Sakrison, M. I. T. (Consultant)
B. J. Schafer
D. I. Schneider, M. I. T. (Consultant)
Dr. M. I. Schneider
Dr. F. C. Schweppe
Dr. W. I. Wells
J. B. Williams, Jr.
J. M. Winett
P. E. Wood, Jr.

Abstracts of the recent publications* of this research program follow, grouped according to the research areas discussed above.

J. F. Nolan

* The listed publications include Technical Reports (TR-), Journal Articles (JA-), Meeting Speeches (MS-), and Group Reports (22G-). Meeting Speeches are listed for information only; no copies of these are available for distribution.

ESTIMATION AND CONTROL THEORY

Stochastic Approximation and "Minimax" Problems
L. A. Gardner, Jr.
11 April 1960
Technical Report 219

A current area in systems research is the study of feedback techniques which automatically adjust system parameters to achieve an ideal system performance. The response of a system to unpredictable inputs is a statistical quantity whose behavior generally varies with the values of the controllable parameters. A criterion for optimum performance, which is often a matter of choice, is consequently formulated in statistical terms. Well-known cases in point are minimum mean-square deviation of output from zero, and maximum average output. A procedure that makes system parameter adjustments in accordance with responses at its current parameter settings may be called an adaptive process. Such a procedure generates increasingly accurate estimates of the optimum parameter values as system operation continues.

A large class of adaptive processes is based on the familiar idea of functional iteration. The title "stochastic approximation" is introduced because the iteration is perturbed by noise. This technique has been investigated in very general terms, but detailed consideration has been confined to criteria that involve regression functions. There is no apparent reason for this limitation. If, for instance, the measure of performance is an error, it may be desirable to operate at the parameter values that minimize the largest possible error. A stochastic approximation procedure appropriate to this type of criterion is the subject matter of this report. We are concerned, therefore, with estimating the minimum or maximum of an extreme value function (and not a mean value function). This is the sense of the word "minimax" in the title. Here we are interested only in proving probability-one convergence of the scheme under certain general restrictions. The efficiency question and the asymptotic distribution of the approximation will be considered at a later date. Moreover, when the iteration scheme is appropriately based upon samples from a stationary time series, it will provide a convergent sequence of approximations to the frequency with maximum power. This will follow from our results, together with those from the theory of consistent spectral estimation.

* * *

Optimum Incorporation of External Observations
with a Marine Inertial Navigation System
D. J. Sakrison
30 November 1962
Technical Report 289

When a marine vessel is used as a platform for tracking space vehicles, the accuracy of the tracking is critically limited by errors in the ship's estimated position and orientation. These reference data are usually estimated by combining the output of an inertial navigation system with additional external position, velocity and/or orientation measurements. To meet stringent accuracy requirements for this estimation, optimum statistical computation procedures for combining the data are required.

This report develops a discrete-time, linear model for the error propagation in an inertial navigation system. This model makes it possible to derive an optimum recursive data processing procedure for combining the inertial system output with the external fixes. A recursive formula for the minimum-mean-square error in estimated position, velocity and orientation is also obtained. The various formulas are directly applicable to broad classes of inertial navigation systems and external measurements. A general discussion is provided on the computational problems associated with both actual data processing and the performance of system-error analyses.

* * *

A Versatile Stochastic Model of a Function
of Unknown and Time Varying Form
H. J. Kushner
J. Math. Anal. Appl. 5, 150 (1962)
Journal Article 1817

Properties of a random walk model of an unknown function are studied. The model is suitable for use in the following problem (among others). Given a system with a performance function of unknown, time varying and possibly multipeak form (with respect to a single system parameter), and given that the only information available are noise perturbed samples of the function at selected parameter settings, then determine the successive parameter settings such that the sum of the values of the observations is maximum. An attempt to avoid the optimal search problem through the use of several intuitively reasonable heuristics is presented.

* * *

Time Optimal Control for Plants
with Numerator Dynamics
M. Athanassiades (Athans) and P. L. Falb
Trans. IRE, PGAC AC-7, 47 (1962)
Journal Article 1909

In this letter, the problem of time optimal control for plants with numerator dynamics (i.e., plants whose transfer functions have zeros as well as poles) is discussed. Under the assumption that the magnitude of the control is bounded, a necessary condition, in the form of a linear differential equation which must be satisfied by the optimal controls, is obtained through the use of the Maximum Principle of Pontryagin. An example which shows that pure relay (or bang-bang) control does not lead to minimum time response is also presented.

* * *

Optimal Control for Linear Time-Invariant Plants
with Time, Fuel and Energy Constraints
M. Athans
Applications and Industry,
No. 64, 321 (1963)
Journal Article 1923

The structure of optimum control systems and the form of the optimal control signals are discussed for minimum time, minimum fuel, and minimum energy control of linear time-invariant

plants. If the control variables are bounded by 1 in magnitude, then the control signals must have the values +1 or −1 for minimum time response; +1, 0, or −1 for minimum fuel operation; +1, −1 or be linearly related to the state of the plant, between these limits, for minimum energy operation.

* * *

Optimal Stochastic Control
H. J. Kushner
Trans. IRE, PGAC AC-7, 120 (1962)
Journal Article 1982

The theory of optimal control of dynamical systems has been the object of much recent attention, and a great deal of insight has been gained about the nature of the problems and solutions. The study of the control of dynamical systems with stochastic elements is in a less fortunate position. Several interesting results have been obtained for certain classes of systems, but so far there does not seem to be a general theory of analytic solution such as exists in the non-stochastic (deterministic) case. In this paper, we contribute towards a general theory and use the results to solve some special cases. Some of the results overlap those published in England by Florentin, but their inclusion is required by the importance of the material as well as the almost complete unfamiliarity in this country with the results previously referred to.

We consider the n-vector system

$$\dot{x} = g(x, u, t) + \alpha(x, u, t) \quad , \tag{1}$$

where α is either a white Gaussian or a Poisson process. The control is u, an r-vector constrained to lie in a space Ω. The function g is differentiable in all its arguments. The problem is to minimize the expected value of the bracketed term in Eq. (2).

$$V(x, t) = \min_{u \in \Omega} E\left[\int_t^T k(x, u, \tau)\, d\tau + k_1(x(T))\right] \quad . \tag{2}$$

The model of Eq. (1) is quite versatile, and an important class of problems fitting the model is considered. The principle of optimality is used to derive a second-order partial differential equation that the optimum control must satisfy, if it exists. A maximum principle also obtains here.

* * *

On the Minimum a Priori Information Necessary
to Construct Adaptive Filters for Stationary Time Series
H. J. Kushner
Trans. IRE, PGAC (accepted for publication)
Journal Article 2057

Studies of various methods for the design of adaptive filters and predictors for stationary time series have been increasing in recent years, and it is of some interest to consider the minimum amount of a priori information that is necessary in principle, in order to build such

adaptive devices. Let s(t) be stationary and independent random functions with the rational, but incompletely known, spectral densities

$$S_s(\omega) = c_1^2 \prod_1^r (\omega^2 + \alpha_i^2) \Big/ \prod_1^p (\omega^2 + a_i^2) \tag{1a}$$

$$S_n(\omega) = c_2^2 \prod_1^t (\omega^2 + \beta_i^2) \Big/ \prod_1^q (\omega^2 + b_i^2) \tag{1b}$$

where $r < p$ and $t < q$. Let the <u>only</u> <u>observable</u> <u>quantity</u> be $y(t) = s(t) + n(t)$

$$S_y(\omega) = S_s(\omega) + S_n(\omega) = c^2 \prod_1^k (\omega^2 + \delta_i^2) \Big/ \prod_1^h (\omega^2 + d_i^2) \quad . \tag{2}$$

We will be concerned with the following question. What is the minimum amount of <u>a priori</u> information on the statistical characteristics of s(t) and n(t) that is necessary in order to build an adaptive linear (in y(t)) least-squared estimator of s(t)? The design of the filter and the process of adaptation can be based only on the <u>a priori</u> given information and the information contained in y(t).

All the information that is necessary to determine the optimum linear filter is contained in the unknown functions $S_s(\omega)$ and $S_n(\omega)$. In fact, the optimum infinite lag filter is[1]

$$H_\infty(\omega) = S_s(\omega)/(S_n(\omega) + S_s(\omega)) \quad ; \tag{3}$$

$S_y(\omega)$ is computable from, and contains all the information in, y(t); hence, the above question may be posed as: what is the minimum <u>a priori</u> information that is necessary to deduce $S_n(\omega)$ and $S_s(\omega)$ uniquely from $S_y(\omega)$?[2] In the sequel we will be concerned with the conditions on $S_n(\omega)$ and $S_s(\omega)$ that allow a unique factorization of $S_y(\omega)$ into a sum of the forms (1a) and (1b). This problem may be viewed as the uniqueness of the recombination, into the forms (1a) and (1b), of the partial fraction expansion of the sum of (1a) and (1b). It is always assumed that, if the pair $(S_s(\omega), S_n(\omega))$ were given, one has enough information to pick out the member of the pair that corresponds to s(t). For the sake of exposition only, we assume that the d_i^2 of (2) are real and distinct.[3]

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1. **The form (3) is given merely to indicate the typical character of the dependence of the form of the optimum filter upon $S_s(\omega)$ and $S_n(\omega)$. These two quantities must always be determined, regardless of the lag of the filter.**

2. **There are many ways of phrasing the question: for example, the adaptive design could be based on the estimation of the correlation function of y(t). However, all approaches are equivalent to the above approach in the sense that, if the filter cannot be designed by using estimates of $S_y(\omega)$, it cannot be designed at all. Actually, only $S_s(\omega)/S_n(\omega)$ is necessary, but the ratio cannot be computed unless both $S_s(\omega)$ and $S_n(\omega)$ can be computed.**

3. **If the d_i^2 are multiple or occur in complex conjugate pairs, the general conclusions of the paper are still valid.**

Time-, Fuel-, and Energy-Optimal Control
of Nonlinear Norm-Invariant Systems
M. Athans, P. L. Falb and R. T. Lacoss
Trans. IEEE, PTGAC (accepted for publication)
Journal Article 2060

Nonlinear systems of the form $\dot{\underline{x}}(t) = \underline{g}\,[\underline{x}(t); t] + \underline{u}(t)$, where $\underline{x}(t)$, $\underline{u}(t)$, and $\underline{g}\,[\underline{x}(t); t]$ are n-vectors, are examined in this paper. It is shown that: If $||\underline{x}(t)|| = \sqrt{x_1^2(t) + \ldots + x_n^2(t)}$ is constant along trajectories of the homogeneous system $\dot{\underline{x}}(t) = \underline{g}\,[\underline{x}(t); t]$ and if the control $\underline{u}(t)$ is constrained to lie within a sphere of radius M, i.e., $||\underline{u}(t)|| \leq M$ for all t, then the control $\underline{u}^*(t) = -M\,\underline{x}(t)/||\underline{x}(t)||$ drives any initial state $\underline{\xi}$ to $\underline{0}$ in minimum-time and with minimum-fuel, where the consumed fuel is measured by $\int_0^T ||\underline{u}(t)||\,dt$. Moreover, for a given response time T, the control $\tilde{\underline{u}}(t) = -||\underline{\xi}||\,\underline{x}(t)/T\,||\underline{x}(t)||$ drives $\underline{\xi}$ to $\underline{0}$ and minimizes the energy measured by $\frac{1}{2}\int_0^T ||\underline{u}(t)||^2\,dt$. The theory is applied to the problem of reducing the angular velocities of a tumbling asymmetrical space body to zero.

* * *

Adaptive Predictors
L. A. Gardner, Jr.
Trans. of the Third Prague Conference
on Information Theory, Statistical Decision Functions
and Random Processes
Meeting Speech 625

Prediction theory has become a well-known discipline to the mathematical engineer and now plays a significant role in the analysis and design of many types of physical systems. However, the supposition made for the purposes of analysis that process statistics are completely known is often an unrealistic assumption. There is, therefore, the important problem of constructing the optimum predictor from a single process realization in the absence of prior quantitative knowledge.

The purpose of this report is a rigorous study of three methods for estimating the linear least-squares predictor of an observable time series. The realization is assumed to be some member of a class of one-sided linear processes (i.e., the output of a discrete linear system, with summable impulse response coefficients, driven by pure white noise). Each of the techniques produces an estimate of the optimum predictor calculated in a sequential fashion as more observations become available. A measure of "relative asymptotic adaptive efficiency" is introduced as the record length increases without bound. The methods are compared on the basis of it, as well as the total elapsed time required to compute the estimate. The advent of high-speed computers makes the general results of practical as well as theoretical interest.

The central problem is one in statistical estimation, and hence the terminology and theoretical tools used in analyzing the procedures are necessarily different from those usually found in engineering treatments of related subjects. There is a self-contained summary which can be read independently of the body of the text.

* * *

Stochastic Approximation and Its Application to Problems
of Prediction and Control Synthesis
L. A. Gardner, Jr.
International Symposium on Nonlinear Differential
Equations and Nonlinear Mechanics
Academic Press, New York (1963), p. 241
Meeting Speech 170A

Although the technique of Stochastic Approximation was introduced and developed by statisticians there is an increasing interest in the subject on the part of those concerned with automatic control and related topics. The purpose of this paper is to give a brief exposition of the technique, with emphasis on certain aspects, and to show how the idea is applicable to a simple control problem. Section 1 introduces the subject matter with the now "classical" Robbins-Munro and Kiefer-Wolfowitz processes. Dvoretzky's general Stochastic Approximation scheme (a noisy functional iteration for a fixed point) is presented, and the convergence criteria are discussed from the viewpoint of the design of nonparametric procedures. As an example, it is shown how to construct the least-squares infinite memory predictor of a weakly stationary time series with only qualitative restriction on second- and fourth-order moments. Section 2, which can be read independently of Sec. 1, contains an ad hoc treatment of a problem in stochastic control. The first-order control laws considered constitute a one parameter family. For a certain special and important case of the model, the limiting distribution of the controlled output is derived as a function of the aforementioned parameter. A comparison is also made with an ideal first-order control.

* * *

A Simple Iterative Procedure for the Identification
of the Unknown Parameters of a Linear Time Varying Discrete System
H. J. Kushner
Joint Automatic Control Conference, New York (June 1962)
J. of Basic Engineering (ASME) (accepted for publication)
Meeting Speech 415

A new approach to the "adaptive control system" problem of the determination of the process dynamics of a time varying system is described. The unknown parameters are the parameters of the impulse response of a linear discrete system. The identification procedure is a first-order iteration and is designed to operate with the natural inputs of the system to be identified. After each new (single) input, new estimates of all the unknown parameters are computed.

The method is computationally simple and, in its analysis, the effects of additive noise in the observations (of both input and output), random drift with time or neglected parameters of the impulse response are handled with relative ease. Time variations are taken directly into account, thus eliminating the necessity of the usual assumption of stationarity over a period of time.

Let the response of the unknown system ($\underline{\theta}$) to any input sequence u_n, be $r_n = \sum_{i=0}^{h-1} \theta_i u_{n-1}$. The estimate (model) of the system parameter θ_i at time n is $x_i(n)$ and the output of the model

is $\bar{r}_n = \sum_0^{m-1} x_i(n)\, u_{n-i}$. It is not assumed, in general, that $h = m$, since some of the parameters of the system may be neglected either by choice (for computational convenience since, in reality, h may be infinite) or due to ignorance of their existence. These neglected parameters have important effects.

The adjustments of the estimate $x_i(n)$ are based on measurements of the first differences of the error function $y_n = 1/2\,(r_n - \bar{r}_n)^2$ (although the methods discussed are not restricted to the use of this error function). Let us define the first difference, with respect to the quantity $x_i(n)$, as $y_i^c(n)$, $(i = 0, \ldots, m-1)$,

$$y_i^c(n) = \frac{1}{2c}\left(y_n(x_i(n) + c) - y_n(x_i(n) - c)\right)$$

$$= u_{n-i}(r_n - \bar{r}_n) \quad .$$

Defining the m-vectors whose i^{th} $(i = 0, \ldots, m-1)$ components are $x_i(n)$, θ_i, and $y_i^c(n)$ as $\underline{x}_n$, $\underline{\theta}$, and $\underline{y}_n^c$ respectively, the basic iterative procedure may be written as

$$\underline{x}_{n+1} = \underline{x}_n - a_n \underline{y}_n^c \quad .$$

If $h = m$, the convergence to $\underline{\theta}$ is exponential under very wide conditions on u_n. The setting $a_n = (\sum_0^{m-1} u_{n-i}^2)^{-1}$ yields the most rapid stepwise convergence. It is not necessary that the u_n be available for computation. It is only required that they be available to the input to the model. (There is a true distinction since, in many problems, the mechanization and the computation problem will be different in each case.)

In the more general case, when $h > m$ and there exist time variations in $\underline{\theta}$ and errors in the measurements of the inputs and outputs, $\underline{x}_n$ converges to a random variable whose moments are calculable under wide conditions on the errors and inputs. The greatest part of the paper is taken up with an analysis of the effects of the errors on the convergence properties. A geometric interpretation is given. Under certain restrictions on the input, the equation of the variance (or covariance matrix) of the error is derived and, from this, one may determine the value of a_n which minimizes the variance. For the case of no time variations, the method is modified such that the variance of the error goes to zero (at rate $1/n$ in most circumstances).

For the most part the inputs are assumed to be random (either correlated or uncorrelated). Although the assumption of statistical regularities on the inputs of most real systems is rather strong, it is useful as a means of obtaining insights into the behavior with arbitrary inputs which could not otherwise be obtained.

* * *

Hill Climbing Methods for the Optimization
of Multi-Parameter Noise Disturbed Systems
H. J. Kushner
Joint Automatic Control Conference, New York (June 1962)
J. of Basic Engineering (ASME) (accepted for publication)
Meeting Speech 549

Modifications of the statistical technique of Stochastic Approximation are used as a means of experimentally optimizing multiparameter systems (maximizing expected performance over the parameter space) on which the only available information (aside from minor mathematical restrictions) is obtainable from noise perturbed samples of the performance.

Both Gradient and Relaxation processes are considered and methods are given for experimentally adjusting certain undetermined process constants for most rapid (mean square) convergence to the optimum parameter setting.

The problem of efficiency of use is more acute in spaces of high dimensionality, and then, without any additional a priori knowledge, it is desirable that the relative importance of the parameters be taken into account in the design of the process. It is shown that this can be efficiently effected by the use of a series of truncated one dimensional processes along selected lines in parameter space. Theoretical results on several methods of selecting these lines and the lengths of the truncated processes are given.

Random Search procedures are investigated and shown to be limited to the few observations necessary to locate a good starting point for the iterative procedures. Results of computer simulations with as many as 18 parameters are presented.

* * *

Minimum-Fuel Feedback Control Systems:
Second-Order Case
M. Athans
AIEE Fall General Meeting, Chicago (7-9 October 1962)
Trans. AIEE (accepted for publication)
Meeting Speech 615

The control u(t), $|u(t)| \leq 1$, is determined for the control of a linear plant G(s) from an initial state to a terminal state, such that the fuel $F = \int_0^{t_f} |u(t)|\, dt$ is minimum. The Maximum Principle is used to prove that the optimal u(t) is necessarily piecewise constant and that u(t) = +1, 0, or −1. The optimal feedback control function is derived for the plants $G(s) = 1/s^2$ and $G(s) = 1/(s+1)(s+2)$ using the concept of the iso-fuel curves. The phase plane is divided into three regions of operation such that to each region corresponds a value for the control u(t).

* * *

Iterative Control of an Undamped Harmonic Oscillator in the Presence of Unknown Noise
D. J. Sakrison
Meeting on Recent Advances in Nonlinear Stochastic Problems
University of California, Los Angeles (9-20 July 1962)
Meeting Speech 635

We are concerned with the following problem. Given a harmonic oscillator described by the differential equation

$$\frac{d^2x}{dt^2} + \omega^2 x = u(t) \quad , \tag{1}$$

we wish to choose a control law u(t) to drive the system to the (unstable) equilibrium state $x = \dot{x} = 0$ when we are able to observe only

$$z_1(t) = x(t) + \epsilon_1(t) \quad \text{and} \quad z_2(t) = \frac{1}{\omega}\dot{x}(t) + \epsilon_2(t) \quad , \tag{2}$$

in which $\epsilon_1(t)$ and $\epsilon_2(t)$ are stochastic noise processes.

The control law to be studied here is patterned directly after the Robbins-Munro procedure for locating the root of a regression function by observing a sequence of different realizations of the random variable involved. We will periodically observe $z_1(t)$ and $z_2(t)$ every T seconds. After each observation, we apply a control force of magnitude

$$u_n = -a_n\,[z_1(nT) + bz_2(nT)] \tag{3}$$

for a duration of T_p sec, $0 < T_p \leqslant T$. In this expression, n denotes the fact that u_n is the n^{th} control force applied in our sequence; our time reference is governed by assuming that we first observed the process and applied a control force at t = T.

Let us represent the noise processes by the column vector

$$\underline{\epsilon}(t) = \begin{bmatrix} \epsilon_1(t) \\ \epsilon_2(t) \end{bmatrix} \quad .$$

We assume only that $\underline{\epsilon}(t)$ has zero mean; that $\underline{\epsilon}_{kT}$ and $\underline{\epsilon}_{NT}$ are statistically independent for $n \neq k$; and that

$$E\,\{||\underline{\epsilon}_t||^2\} \leqslant \sigma^2 < \infty \quad \text{for all } t \quad . \tag{4}$$

Thus the explicit statistical behavior of $\underline{\epsilon}(t)$ is <u>not</u> assumed to be known. We do not assume that $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are statistically independent random variables.

We will demonstrate that for a suitable choice of a_n and b in the control law,

$$E\,\{||\underline{x}_t||^2\} \rightarrow 0 \quad \text{as} \quad t \rightarrow \infty$$

in which $\underline{x}(t)$ denotes

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} x(t) \\ \frac{1}{\omega}\dot{x}(t) \end{bmatrix} . \tag{5}$$

* * *

Time-Optimal Velocity Control
of a Spinning Space Body
M. Athans, P. L. Falb and R. T. Lacoss
AIEE Winter General Meeting, New York
(27 January - 1 February 1963)
Trans. IEEE (accepted for publication)
Meeting Speech 703

The problem of minimum-time angular velocity control of a spinning space body, with a single axis of symmetry, is examined. The control thrusts are assumed bounded. Two-axis, gimballed, and single-axis control are considered. The optimal control law is derived for each method. Comparison of the response times indicates that gimballed control is best.

* * *

A New Method of Locating the Maximum Point
of an Arbitrary Multipeak Curve in the Presence of Noise
H. J. Kushner
Joint Automatic Control Conference, University
of Minnesota, Minneapolis, Minnesota
(19-21 June 1963) (accepted for presentation)
J. of Basic Engineering (ASME) (accepted for publication)
Meeting Speech 707

A versatile and practical method of searching a parameter space is presented. Theoretical and experimental results illustrate the usefulness of the method for such problems as the experimental optimization of the performance of a system with a very general multipeak performance function when the only available information is noise-disturbed samples of the function. At present, its usefulness is restricted to optimization with respect to one system parameter. The observations are taken sequentially; but, as opposed to the gradient method, the observation may be located anywhere on the parameter interval. A sequence of estimates of the location of the curve maximum is generated. The location of the next observation may be interpreted as the location of the most likely competitor (with the current best estimate) for the location of the curve maximum. A Brownian motion stochastic process is selected as a model for the unknown function, and the observations are interpreted with respect to the model. The model gives the results a simple intuitive interpretation and allows the use of simple but efficient sampling procedures. The resulting process possesses some powerful convergence properties in the presence of noise; it is nonparametric and, despite its generality, is efficient in the use of observations. The approach seems quite promising as a solution to many of the problems of experimental system optimization.

On Optimal Control of Self-Adjoint Systems
M. Athanassiades (Athans), P. L. Falb and R. T. Lacoss
Joint Automatic Control Conference,
University of Minnesota, Minneapolis, Minnesota
(19-21 June 1963) (accepted for presentation)
Trans. IEEE (accepted for publication)
Meeting Speech 709

Given the system $\dot{\underline{x}}(t) = \underline{A}(t)\,\underline{x}(t) + \underline{u}(t)$, with $\underline{A}(t) = -\underline{A}'(t)$ and $||\underline{u}(t)|| \leqslant 1$, it will be shown that the control $\underline{u} = -\underline{x}(t)/||\underline{x}(t)||$ drives any initial state to zero in such a manner that the response time, the consumed fuel, and a linear combination of time and control energy are minimized. The theory is applied to the optimum angular velocity control of a tumbling space body.

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Minimum-Fuel Control of Second-Order Systems
with Real Poles
M. Athanassiades (Athans)
Joint Automatic Control Conference, University
of Minnesota, Minneapolis, Minnesota
(19-21 June 1963) (accepted for presentation)
Trans. IEEE (accepted for publication)
Meeting Speech 710

The controlled system has the transfer function $G(s) = K/(s-\lambda_1)(s-\lambda_2)$. The control input is $u(t)$, $|u(t)| \leqslant 1$, and the output is $y_1(t)$. The control which forces any initial state $y_1(0)$, $\dot{y}_1(0)$ to the terminal state $y_1(T)$, $\dot{y}_1(T)$, such that $-K/\lambda_1\lambda_2 \leqslant y_1(T) \leqslant K/\lambda_1\lambda_2$, $\dot{y}_1(T) = 0$, and which minimizes the fuel $F(T) = \int_0^T |u(t)|\,dt$ is determined. The phase plane is divided into three sets G_-, G_+, and G_o; if the state is in G_-, then $u(t) = -1$ is used; if the state is in G_+, then $u(t) = +1$ is used; if the state is in G_o, then $u(t) = 0$ is used.

* * *

Methods for the Adaptive Optimization
of Binary Detection Systems
H. J. Kushner
IEEE International Convention, New York
(25-28 March 1963)
1963 IEEE International Convention Record
Meeting Speech 732

Adaptive techniques are investigated for the problem of the experimental optimization of maximum signal-to-noise ratio binary detectors which operate in a partially unknown environment; e.g., unknown transmission path or noise correlation function. Two approaches are taken and compared in detail. The first involves the use of statistical estimates of the unknown quantities that are obtained by the appropriate averaging operations. The second, and more interesting, approach makes use of the methods of stochastic approximation to develop a convergent iterative process on the parameters of the adaptive filter. The iterates converge to the optimum values in mean square and with probability one. Let $\underline{x}$ be the filter parameter vector. The

asymptotic distribution of $\sqrt{n}(\underline{x}_n - \underline{x}_{opt.})$ is given, and the values of the risk function En(S/N(opt) − S/N(n)) are given for both cases. Since the latter approach requires observations of the output of the filter only, and does not involve the estimation of the unknown characteristics of the signal or noise, it promises to be the more useful in many practical situations. The following situations are considered: (1) unknown discrete signal and known noise correlation; (2) unknown noise correlation and known discrete signal; (3) the continuous versions of (1) and (2); (4) a signal optimization problem with unknown noise correlation. The problem of unknown epoch is briefly considered and an example is given to illustrate the fact that, if either approach is not carefully used when the epoch is unknown, convergence to a nonoptimum system may take place.

* * *

Optimal Control for Norm Invariant Systems
M. Athans
Computation Laboratory Seminar Harvard University
(17 January 1963)
Meeting Speech 780

Nonlinear systems of the form $\dot{\underline{x}}(t) = \underline{g}\,[\underline{x}(t);t] + \underline{u}(t)$, where $\underline{x}(t)$, $\underline{u}(t)$, and $\underline{g}\,[\underline{x}(t);t]$ are n-vectors, are examined in this paper. It is shown that: if $||\underline{x}(t)|| = \sqrt{x_1^2(t) + \ldots + x_n^2(t)}$ is constant along trajectories of the homogeneous system $\dot{\underline{x}}(t) = \underline{g}\,[\underline{x}(t);t]$ and if the control $\underline{u}(t)$ is constrained to lie within a sphere of radius M, i.e., $||\underline{u}(t)|| \leqslant M$, for all t, then the control $\underline{u}^*(t) = -M\underline{x}(t)/||\underline{x}(t)||$ drives any initial state $\underline{\xi}$ to $\underline{0}$ in minimum-time and with minimum-fuel, where the consumed fuel is measured by $\int_0^T ||\underline{u}(t)||\,dt$. Moreover, for a given response time T, the control $\tilde{\underline{u}}(t) = -||\underline{\xi}||\,\underline{x}(t)/T\,||\underline{x}(t)||$ drives $\underline{\xi}$ to $\underline{0}$ and minimizes the energy measured by $\frac{1}{2}\int_0^T ||\underline{u}(t)||^2\,dt$. The theory is applied to the problem of reducing the angular velocities of a tumbling asymmetrical space body to zero.

* * *

An Efficient Iterative Method
for Optimizing System Performance
H. J. Kushner
13 April 1961
G-Report 22G-0050

A problem in systems design is the determination of the value of a parameter at which a system has the best performance. The form of the dependence of the performance function on the parameter may be unknown and, in addition, the performance may be partly determined by random variables. The optimization problem is rather difficult under these conditions and it is often necessary to resort to "adaptive system" techniques. An iterative procedure is described for determining the parameter value at which the system has its best average performance. It is based only on experimental observations of the systems operation. The basic procedure is one described by Kiefer and Wolfowitz for locating the extremum of an unknown regression function.

The procedure referenced contains several undetermined constants and sequences. The efficiency, or mean square rate of convergence, of the iterates to the optimum value depends upon these undetermined numbers. A logic is presented for their calculation from observational data only such that the resulting rate of optimization (adaptation) is highest, both initially and asymptotically. The result is an adaptive process that adjusts itself to the system being optimized during the course of the optimization, thus increasing the rate of performance improvement.

Theoretical and experimental results are very close.

DIGITAL SYSTEM DESIGN

Design Using Computers
T. C. Bartee
In A Survey of Switching Circuit Theory
Edited by T. C. Bartee and E. J. McCluskey
McGraw-Hill, New York (1962)
Journal Article 1665

The use of Boolean algebra facilitates the analysis and synthesis of networks composed of switching elements. The utility of this approach is based on the isomorphic relation between the operation of a given switching circuit and the Boolean expression for the circuit. This parallelism has brought about a widespread use of algebraic techniques in the design of digital machines. Since the network to be analyzed or synthesized will physically realize the Boolean expression for the network, it is possible to construct a model of a machine using Boolean expressions and, in effect, to study and simulate the operation of the machine using the expressions instead of a physical model. Furthermore, algebraic manipulations may be performed on the expressions for a given network, and a reduction in the length of the expressions will yield a corresponding reduction in the number of network components required to physically realize the expression.

The reason for using digital computers to design logical networks should be fairly apparent. As the number of different input variables in Boolean expressions increases, the amount of computation required to minimize the expressions increases on the average, exponentially. Therefore, designing networks of even modest size often calls for considerable labor on the part of the logical designer and a corresponding delay in the design process. By using a digital computer to perform the straightforward but often tedious and time-consuming calculations, the design of digital machines can be facilitated.

This paper will be concerned with a specific set of programs, those prepared at Lincoln Laboratory for the IBM 7090 general-purpose digital computer. The presentation will be limited to those programs which are used to design two-level combinational networks. Programs have also been written for sequential circuit design, specifically the memory assignment problem.

* * *

Theory and Design of Digital Machines
T. C. Bartee, I. L. Lebow and I. S. Reed
McGraw-Hill – Lincoln Laboratory Publication Series, New York (1962)

This book presents an advanced discussion on the theory underlying the design of modern digital machines. It utilizes a systematic and integrated approach first formulated in the Lincoln Laboratory of the Massachusetts Institute of Technology and used to design numerous machines in the Laboratory. One aspect of this approach is the combining of machine design and switching theory.

Distinctive features of the book, in addition to its systematic approach, are its logical organization and depth and breadth of treatment of the design of digital machines. The authors have compiled an up-to-date book, containing much material heretofore not published. Questions

follow each chapter. Both special-purpose and general-purpose machines are covered within the same framework.

The book also contains an introduction to the material generally coming under the category of "switching circuits." The treatment contains both the theory of combinational and sequential machine design. The book does not contain a description of circuit or component design, but treats machine design from a general viewpoint. Included in the discussion are the algebraic and logical foundations fundamental to the theoretical and practical aspects of digital machine design.

* * *

The Automatic Design of Logical Networks
T. C. Bartee
3 December 1958
Technical Report 191

The operation of a logical network can be described by means of a set of Boolean expressions which represent, in symbolic form, the relationship between the inputs and outputs of the network. As the number of inputs and outputs to the network increases, however, the complexity of these equations increases significantly and the amount of calculation required to solve certain problems becomes prohibitive. In order to facilitate the design of large networks, a set of computer programs has been prepared which generates the expressions that describe the input-output relationships for given problems, and then automatically minimizes these expressions, presenting the logical designer with a set of expressions which can be used to design the required network most efficiently.

This design technique makes possible the construction of logical networks which perform such functions as sine, arc sine and square root. When these networks are used as a part of the arithmetic element of a digital computer, these functions may be yielded by a single instruction, significantly reducing the time required to solve certain problems. The latter section of this report describes a sine instruction which replaces an angle with the sine of that angle, in less than 10 μsec.

* * *

State Assignment Algorithm for Clocked Sequential Machines*
M. I. Schneider
25 May 1962
Technical Report 270

The number of gating elements in a sequential circuit, and therefore its cost and complexity, is dependent upon the assignment of codes to the sequential-circuit memory elements. This report presents an algorithm (which has been programmed) for assigning codes to the states of a clocked sequential circuit in a manner that permits the circuit to be efficiently realized. A

***This report is based on a thesis of the same title submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Division of Engineering and Applied Physics at Harvard University.**

procedure that employs the state assignment algorithm for the logical design of the control logic of fixed-program computers is proposed in which the computer control logic is organized as a sequential circuit.

* * *

Automatic Design of Logical Networks
T. C. Bartee
1959 WESCON Proc., San Francisco (March 1959)
Journal Article 1139

The frequent use of transcendental functions in digital computer programs has presented a basic problem in the design of real-time control systems. Since there are usually no single instructions to perform such functions as sine χ, arc sine χ, etc., two other techniques have been used.

The first technique involves storing a table of values for the function in rapid access memory. The advantage of this technique lies in the speed obtainable; however, in order to provide reasonable accuracy, the table must be of considerable length. Computations which involve precise physical measurements such as those made by one of the newer radars would require a table which would occupy a significant portion of most storage devices. An alternate technique involves storing a smaller table, and then interpolating between the values stored in this table. The principal advantage of table storage is then lessened for the interpolation routine requires time and the function can no longer be generated as quickly.

The second technique involves the use of a programmed approximation routine. Most of the routines used are based on polynomial approximations which require several multiplications, and as a consequence, are time-consuming.

The utilization of digital computers in real-time control systems which must process large amounts of data at high speeds has made the problem more acute. Most programs for such systems require values for sines, cosines, etc., quite often, and the necessary computations must be made quickly and with considerable accuracy. The need therefore exists for a very fast and accurate method of generating trigonometric functions.

Lincoln Laboratory has recently prepared a set of flexible computer programs which automatically perform the design of logical networks which will perform such functions in a single step. These networks may then be connected into the arithmetic element of an internal-binary-operation digital computer, providing the machine with instructions which will yield the sine, arc sine, etc., of an angle stored in one of the registers of the arithmetic element. The design technique yields a logical network with a number of input and output lines, the input lines representing the independent variable and the output lines the dependent variable. The networks are completely digital in operation, utilizing high-speed transistor and diode logical circuitry.

Because of the magnitude of the design problem for large logical networks, conventional manual design techniques could not be used. For instance, the complete set of expressions needed to describe a 12-input bit and 14-output bit sine network in developed normal form contains approximately 90,000 symbols. Normal design techniques involving manual calculations

would prohibit either the generation or simplification of a set of expressions of this complexity. It was necessary, therefore, to design and prepare a set of digital computer programs which would automatically generate the desired set of expressions, simplify them, and then check the simplified equations. The series of programs written are general purpose in design and may be used to generate and simplify the logical equations describing any function with a unique value of the dependent variable for each value of the independent variable.

* * *

Automatic Synthesis of Multi-Function Networks
T. C. Bartee
AIEE Fall General Meeting, Chicago (11-16 October 1959).
Journal Article 1295

A set of programs has been written for a general-purpose computer which automatically synthesizes the design expressions for logical networks. These programs first generate the expressions describing the desired network, and then simplify these expressions automatically. The computer printout details a minimal two-level network.

Utilizing an array of photorectifiers, the active elements of which are determined by holes punched in a standard 12-row by 80-column card, a multifunction network has been constructed which yields outputs which are selected transformations of the input to the network. The transformation which the network performs may be changed by changing the punched card which controls the logic of the net.

The pattern of holes which is punched into the cards is determined by the computer printouts from the logical design programs. This technique permits the construction of special-purpose high-speed instructions which can be selected according to the particular programming needs.

* * *

Computer Design of Multiple-Output Logical Networks
T. C. Bartee
Trans. IRE, PGEC EC-10, 21 (1961)
Journal Article 1645

An important step in the design of digital machines lies in the derivation of the Boolean expressions which describe the combinational logical networks in the system. Emphasis is generally placed upon deriving expressions which are minimal according to some criteria. A computer program has been prepared which automatically derives a set of minimal Boolean expressions describing a given logical network with multiple-output lines. The program accepts punched cards listing the in-out relations for the network, and then prints a list of expressions which are minimal according to a selected one of three criteria. This paper describes the basic design procedure and the criteria for minimality.

* * *

Design Using Computers
T. C. Bartee
AIEE General Meeting, New York (February 1960)
Journal Article 1540

A combinational switching circuit is generally synthesized in two steps. First, the input-output relationships are specified, generally by means of a table of combinations, and an expression describing the desired circuit in Boolean algebra derived. Second, the algebraic expression describing the desired circuit is minimized.

The second step, the minimization of a given Boolean expression, has been the classic problem in switching circuit design, and Drs. Tonana and McCluskey have described (in preceding papers) several of the present-day techniques. However, even with these sophisticated and efficient procedures, the number of steps required to minimize expressions in from six to eight variables becomes extremely laborious, and the minimization of expressions in more than eight variables often requires a prohibitive amount of calculation.

As a result, computer programs have been prepared which automatically minimize Boolean expressions. These programs not only reduce the amount of work required in machine design, but also offer the designer a chance to try various configurations without involving large numbers of time-consuming calculations.

There are four categories into which combinational circuits are generally separated: single-output, two-level networks; single-output, multilevel networks; multi-output, two-level networks; and multilevel, multi-output networks. Typical networks of each type will be shown.

The general outline of the Lincoln Laboratory logical design program will be described and typical design expressions for each of the four types of networks presented. The various options offered the designer, such as all irredundant normal forms or a single, minimal normal form, and the use of the programs for multi-output networks will be briefly described.

* * *

An Alpha-State Finite Automaton for Multiplication by Alpha
J. M. Winett
Trans. IRE, PGEC EC-11, 412 (1962)
Journal Article 1924

The problem considered here is to construct a finite automaton which, when presented with an arbitrary number N, will produce as the output αN where α is a given fixed integer. The input number N is assumed to be represented in binary form with the least significant digit a_0 presented first. The output will also be a binary number which replaces the input number.

$$N = a_0 a_1 a_2 \ldots a_n \qquad \alpha N = b_0 b_1 b_2 \ldots b_n \quad .$$

Recall that a Turing machine is described by a state-symbol table indicating the output, direction, and next state for each state-symbol pair. The Turing machines presented here move in only one direction, reading the input bits in sequence. If, in addition, the present output is only dependent on the present state, then this Turing machine is a finite automaton. We will construct an α-state Turing machine which reads the bits of N and writes the bits of αN. For

α even, this Turing machine is a finite automaton. For α odd, the present output is dependent on both the present state and the present input symbol. For this case, Lee has shown how to construct a finite automaton with $2\alpha + 1$ states, which is equivalent to our α-state Turing machine. Thus the α-state Turing machine which we present can be interpreted in terms of a single input – single output sequential machine.

* * *

Error-Free Threshold Devices with Parameter
and Signal Variation
P. E. Wood
Trans. IEEE, PTGEC (accepted for publication)
Journal Article 1948

The effects of parameter and signal variations on the error properties of threshold devices are studied. Quantitative expressions for the allowed (error-free) variations are derived from a nonideal model of a threshold device. The allowed variations are calculated for threshold functions of four or less variables, and the results are tabulated in a form which allows rapid location of a desired function.

* * *

Computation With Finite Fields
T. C. Bartee, D. I. Schneider
Inform. and Control (accepted for publication)
Journal Article 2012

Recent studies in the area of communications have drawn heavily upon theorems and techniques from modern abstract algebra. For instance, the properties of error-detecting and -correcting block codes are generally proven by arguments based on theorems from linear algebra. Further, the techniques for encoding and decoding are generally stated using the language and symbology of modern algebra. As might be expected, the algorithms which have been invented sometimes cannot be easily implemented in a conventional general-purpose digital computer. For instance, a rather simple special-purpose device, the shift register with feedback, can rapidly perform encoding, decoding, or checking operations which cannot be easily performed by the conventional general-purpose computer.

Among the axiomatic systems which have received special attention are mathematical structures known as Galois or finite fields. For instance, the Bose-Chaudhuri-Hocquenghem error-correcting code, the most efficient multiple-error-correcting block code for independent errors now known, owes its conception primarily to results from this area. Further, the decoding procedure for this code, discovered by Peterson and later generalized by Zierler and Gorenstein, is stated in terms of the theory of finite fields. In constructing an electronic decoder for the Bose-Chaudhuri-Hocquenghem code, we found that by designing the arithmetic unit to perform not conventional binary arithmetic, but instead, Galois field operations, the decoding procedure could be implemented by a rather simple special-purpose digital machine. A similar requirement

for Galois field arithmetic circuits has arisen in some recent applications of linear recurring sequences to space object tracking.

In this paper, we present a technique for (1) systematically generating representations of Galois field elements for a field with a given number of elements, and (2) describing, in a compact closed form, the relations which must be physically realized in order to implement a parallel arithmetic unit which can add, subtract, multiply, and divide, using Galois field elements. Finally, techniques for using a maximal-length linear recurring sequence to modulate a radar transmitter and the means of extracting range information from the returned sequence are described. This involves determining the number of digits separating two n-tuples occurring in a given sequence, and an algorithm which is fast and readily implemented is presented.

* * *

Proposed Symbology for Digital Systems
T. C. Bartee
AIEE National Electronics Conference, Chicago (October 1960)
Proc. AIEE National Electronics Conference
Meeting Speech 52

This report has been prepared as an urgently needed reference on suggested symbols for design and maintenance requirements of digital systems. In four sections, the following topics are presented:

(a) Algebraic symbols for the functional relation between logical variables described by equations,

(b) Drafting symbols for manually prepared logical diagrams,

(c) Printed graphical symbols for logical diagrams prepared by high-speed printer,

(d) Glossary of terms providing essential definitions.

* * *

An Electronic Decoder for Bose-Chaudhuri-Hocquenghem Error-Correcting Codes
T. C. Bartee and D. I. Schneider
Trans. IRE, PGIT IT-8, S-17 (1962)
Meeting Speech 381

This paper describes the design and construction of an electronic error-correcting encoder and decoder for a binary communications channel. The general encoding scheme was published by R. C. Bose and C. R. Ray-Chaudhuri and at about the same time by A. Hocquenghem. The specific code used in the encoder we have constructed adds 35 binary check digits to a given set of 92 binary message digits, forming a 127-binary-digit code word which is then transmitted. The decoder, which is located at the receiver of the communications channel, can correct all combinations of five or fewer errors which occur during transmission of the 127-digit block.

The encoder consists of a 35-stage shift register with feedback and a small control unit. The decoder consists of a 127-stage shift register and a special-purpose digital computer small enough to fit into a desk drawer. Each received block of 127 digits is first stored in the shift

register and then cycled twice through the small digital machine. The first time the machine examines the block and determines whether any errors have occurred; the second time the binary digits which are in error are corrected, or if the received block is not correctable, that is, if the received block is not within distance five of a code word, the machine transmits a signal indicating this.

A received code block containing either five or less errors can be corrected in less than $1/20^{th}$ of a second; blocks containing no errors require substantially less time.

* * *

Decoding of Group Codes for Binary Symmetric Channels*
W. I. Wells
21 March 1960
Group Report 22G-0029

Coding information prior to transmission is a way of inserting redundancy into transmitted sequences so that the effects of noise disturbances during transmission can be minimized at the receiver. Shannon's theorem for noisy channels indicates that with proper encoding and decoding, one can transmit information over a noisy channel with vanishingly small probability of error provided the transmission rate does not exceed the channel capacity. To achieve this goal, the coded sequences must be very long, so that, in effect, they are subjected to the average noisy behavior of the channel rather than to its instantaneous behavior. As a practical matter, the decoding of very long sequences is a severe problem, since for a constant rate of information transmission the number of sequences the receiver must be prepared to decode grows exponentially with the length of the codes.

The investigation of this report considers the decoding problem for an additive (finite field) channel with noise consisting of statistically independent components. The approach is based upon the classification of receivable sequences, such that the complexity of decoding is related more directly to the number of equivalence classes than to the number of receivable sequences. This concept, first studied by Prange, is motivated by the existence of a step-by-step decoding scheme requiring only a knowledge of the "distance" a received sequence is from the set of code sequences. Classification is accomplished by a group of distance-preserving transformations such that all receivable sequences in a given class are an equal distance from the code sequences. It is shown that these classes have the formal properties of complete conjugate classes of finite groups. The theory of group representations is employed to formulate the entire problem and to prove that there do exist complete invariants for the classes, which can be found, in theory at least, by methods of the report. It is shown also that the group of measure preserving transformations, for the case of noise with statistically independent components, is a group of coordinate permutations. Group codes are developed as a special case of the general "equivalence class" technique. This report is concerned primarily with the general formulation of the "equivalence class" approach to decoding rather than with its practical application to specific codes.

*** This report is based on a thesis submitted to the Department of Electrical Engineering at M. I. T. on 26 February 1960, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.**

INFORMATION SYSTEMS

Applications of Boolean Matrices to the Analysis
of Flow Diagrams
R. T. Prosser
22 January 1960
Technical Report 217

An analysis of the structure of flow diagrams, such as those associated with computing machine programs, can be given in terms of Boolean matrices. With each such diagram is associated a pair of Boolean matrices. The first of these, called the connectivity matrix, contains the topological structure of the diagram, and the second, called the precedence matrix, contains its precedence relations. Elementary computations on these matrices are shown to yield detailed information concerning the internal logical consistency of the flow diagram. Possible applications to automatic compiling and debugging procedures are suggested.

* * *

GENDARE (A Generalized Data Reduction System)
M. M. Marill and J. B. Williams
22 January 1962
Technical Report 241

This report contains design specifications for the GENDARE System, a computer programming system that provides data reduction services easily and cheaply when large volumes of serial-access data are to be processed. Development of the GENDARE design was based on recognition and use of the fundamental similarity of all data reduction programs. The system contains: (1) a problem specification language; (2) a library of standardized, building-block routines; (3) a Make-Up Program that translates problem specifications into complete, special-purpose Operating Programs; and (4) Operating Programs that actually perform data reduction by retrieving the data to be reduced, processing it, making dynamic storage assignments and sequencing decisions, and generating requested outputs.

* * *

Studies in Search for a Conscious Evader*
R. Norris
14 September 1962
Technical Report 279

This paper considers a search problem in which the search is directed against a conscious evader or an object controlled by a conscious evader. It is a two-person, zero-sum game called a search evasion game. Although the searcher cannot observe any of the evader's actions, the evader can observe the searcher's and can capitalize on errors that he makes.

At the beginning of the game, the evader hides in one of several boxes. The search process consists of a sequence of looks into the various boxes until the evader is found. Each look into

*** This report is based on a thesis of the same title submitted to the Department of Electrical Engineering at the Massachusetts Institute of Technology on 31 August 1962, in partial fulfillment of the requirements for the degree of Doctor of Science.**

a given box takes a fixed amount of time. If the searcher looks into the box in which the evader is located, he will find the evader with a certain probability – the detection probability associated with the box in question. A particular evasion device is assumed: the evader can move from one box to another between looks. A cost is usually associated with such a move.

Primary emphasis is placed on the study of the search evasion game that involves two boxes, for which solutions have been found. Two limiting forms of the two-box game are considered first. In G^{∞}, moving is prohibited. In G^{O}, the other limiting form, the evader can move at no cost.

The game becomes more interesting when a nonzero but finite cost is associated with each move. In most cases, a finite prohibitive bound on the moving cost exists. When the moving cost exceeds this bound, the searcher's good strategy is identical with his good strategy in G^{∞}. The evader should never move if the searcher uses this strategy. When the moving cost is strictly less than the prohibitive bound, the searcher's good strategy is Markovian in form. That is, the good search strategy can be generated by a finite Markov process in which a look is associated with each transition.

The search evasion game that involves more than two boxes is also studied. In G^{O}, the limiting form in which the moving costs are equal to zero, exact solutions can still be found. The basic properties of the other limiting game, where moving is prohibited, are simple extensions of those that apply when there are only two boxes. In this game, however, the computational effort required to find a solution can be excessive.

The properties of the general many-box game in which the moving costs are neither prohibitive nor equal to zero are quite different from those that apply in the two-box case. Except when the moving costs are very small, the searcher's good strategy can no longer be generated by a Markov process. The complex character of the game is indicated by the partial solution that has been found to the simplest three-box game. The prospects of being able to find exact solutions to the general game in an efficient manner appear to be remote. A particular approach to finding approximately good search strategies is suggested for future research.

* * *

A Data Processing Formalism
A. W. Armenti, B. V. Schafer and J. M. Winett
6 November 1962
Technical Report 283

A great deal of effort has gone into the formulation of programming languages, such as FORTRAN and ALGOL, that provide computer users with a convenient means for communicating with a computer. Languages of this sort are intended primarily as aids for computer users who have standard, numerical problems and are interested in minimizing program preparation time. However, in recent years there has been a growing use of computers in large-scale data processing systems. The programming problems for such systems tend to be nonstandard and, in large measure, nonnumerical. Data storage and retrieval, storage allocation, message translation, and analysis are a few examples of the types of data processing problems that fall into this category.

In the design of such systems, the designers quite often find it as difficult to communicate with one another as with the computer. During the early stages of design, they are frequently called upon to provide descriptions of the computer processes and procedures. At the completion of the system design, they are called upon to provide documentation and specification of the processing. Very little effort has been put into the formulation of a language to serve these purposes. This report presents a language, referred to as the "Formalism," which is designed to meet this need.

The Formalism is intended to serve as an effective vehicle for transmitting computer processing ideas, e.g., program logic and data organization, from one program designer to another. For this reason, while the formal language provides a logical basis for developing efficiently coded forms for an actual program system, it is not meant to be, in its present form, a source language for a compiler.

The Formalism consists of an alphabet of primitive marks or symbols, a set of elementary functions and actions, and formation rules for constructing complex functions and actions. The Formalism assumes a finite memory space consisting of bit locations numbered consecutively from 1 to some chosen limit. Thus, if the memory space has 10,000 bit locations, the Formalism assigns the consecutive integers from 1 to 10,000 to these locations in a 1-to-1 manner giving each bit location a unique address.

The Formalism is machine-independent and can be used to define any process that can be programmed. Whenever specific computer characteristics are explicitly required in the definition of a computer process, e.g., computer word length or the presence of index registers, they can be defined wholly within the Formalism. To describe a particular computer process in which the contents of memory space are changed, expressions called "function definitions," "action definitions," and "set namings," which are involved in the operations to be performed, are defined. An action name together with its variables or arguments forms a "functional statement." The collection of statements listed in the order in which they are to be interpreted is called a "functional program." On suitable interpretation each statement in the functional program results in a change in the memory space, a transfer to some other statement in the functional program, the assignment of arguments to some variables in the functional program, or it results in termination.

* * *

Message Delay in Communication Nets with Storage
L. Kleinrock
Department of Electrical Engineering, M. I. T. (Ph. D Thesis accepted)
McGraw-Hill (accepted for publication)

This thesis investigates the flow of message traffic in communication nets in which there is storage at each of the nodes in the net. The measure of performance used is the average delay experienced by a message as it passes through the net. The results of this study expose the effects of channel capacity assignment, routing procedure, priority discipline, and topological structure on the average message delay, subject to the constraint that the sum of all channel

capacities in the net is a constant. The input traffic is assumed to have Poisson arrival time statistics, with exponentially distributed message lengths. Furthermore, an assumption regarding the independence of the interarrival times and message lengths of the internal traffic statistics is made which simplifies the mathematical analysis; this assumption leads to a model which closely approximates the behavior of the average message delay for nets with dependent traffic (i.e., in the absence of the independence assumption).

Certain new results for simple multiple channel systems indicate that message delay is minimized when the message traffic is clustered into a small number of high capacity channels. The optimum channel capacity assignment (which minimizes an expression for the average message delay) is derived for a communication net with a fixed routing procedure, and subject to the constraint of constant total channel capacity. An analysis for a new delay dependent priority structure is carried out, which provides the system designer with a number of degrees of freedom with which to adjust the relative waiting times for each priority group. Furthermore, a conservation law is developed which allows one to draw a number of general conclusions about the average waiting times for a large class of priority structures.

A class of random routing procedures, described by finite-dimensional, irreducible circulant probability transition matrices is investigated, and the average path length is solved for; a solution for the expected message delay under such routing procedures is also obtained. It is found that random routing results in increased message delay and decreased total traffic handling capability.

A digital network simulation program was written, and its operation is described. The major results from the simulation are summarized below:

(1) The square root channel capacity assignment assigns to each channel enough capacity to handle its average traffic flow, and then assigns the excess capacity in proportion to the square root of the traffic carried by that channel. This assignment results in superior performance as compared to a number of other channel capacity assignments in various nets.

(2) The performance of a straightforward fixed routing procedure, with the square root capacity assignment, surpasses that of simple alternate routing procedures.

(3) Alternate routing procedures adapt the internal traffic flow to suit the capacity assignment (i.e., the bulk of the message traffic is routed to the high capacity channels). This effect is especially noticeable and important in the case of a poor capacity assignment which may come about due to uncertainty or variation in the applied message traffic.

(4) A high degree of nonuniformity in the external traffic matrix results in improved performance for the case of a square root channel capacity assignment.

(5) The quantities essential to the determination of the average message delay are the average path length and the degree to which the traffic flow is clustered. The trade-off between these two quantities allows one to determine the sequence of optimal network topologies which ranges from the star net at small values of network load to the fully connected net as the network load approaches unity.

The generalization of certain theorems, and the relaxation of some of the assumptions are discussed in order to indicate appropriate extensions to this study.

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